Stability Formula for Tetrapods

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1 author:

Kyung-Duck Suh
Seoul National University

Technical Note

Stability Formula for Tetrapods

Kyung-Duck Suh, M.ASCE[1]; and Jin Sung Kang[2]

Abstract: A stability formula has been developed for Tetrapods armoring rubble mound breakwaters based on hydraulic model test results of the present and previous studies. The formula is applicable to breakwaters with various slope angles. It is also applicable to low-crested breakwaters and different packing densities if the corresponding terms are incorporated in the formula. The uncertainty of the formula is also described. **DOI: 10.1061/(ASCE)WW.1943-5460.0000124.** © *2012 American Society of Civil Engineers.*

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Introduction

Tetrapod is one of the armor units used world wide for rubble mound breakwaters. Hudson (1959) proposed a stability formula for Tetrapods based on the results of small-scale model tests using regular waves. Van der Meer (1988) and De Jong (1996) proposed formulas using irregular wave test results for surging and plunging waves, respectively. De Jong's (1996) formula contains additional terms that take into account the influence of crest elevation and packing density. Van der Meer (2000) recommended using these formulas as a set of formulas that intersect at the point of minimum stability. These formulas include the effects of wave period, storm duration, and damage level, which were not included in Hudson's (1959) formula, but they were proposed for a constant structure slope of 1:1.5. In this study, we develop a set of formulas that can be used for various structure slopes based on hydraulic model test results of the present and previous studies.

Hydraulic Model Tests

Tests were carried out in the wave flume at the Hydraulic and Coastal Engineering Laboratory of Seoul National University. Fig. 1 shows the arrangement of the model breakwater and wave gauges. The flume was 36-m long, 1.0-m wide, and 1.2-m deep. It was equipped with a piston-type wave generator at one end and a wave absorber at the other. A horizontal bed with a $1/25$ foreshore slope was installed at the elevation of 20 cm from the bottom of the flume. The breakwater model was placed at a distance of 25 m from the wave maker and a few centimeters from the beginning of the horizontal bed. The test section was divided into two channels by a vertical wall along the wave flume, each having a width of 0.6 and 0.4 m, respectively. The breakwater was installed in the wider channel, and the other channel was left empty. To measure the incident waves, three wave gauges were installed in the empty channel. The free surface displacements measured by these wave gauges were used to separate the incident and reflected waves using the method of Suh et al. (2001). Although the channel is empty, wave reflection occurs from the sloping bed and wave absorber. Waves were generated at a water depth of 0.6 m, and the water depth at the structure was 0.4 m.

The main characteristics of Tetrapods were: height $H_T =$ 6.2 cm; nominal size $D_n = 4.03$ cm; mass density $\rho_a =$ 2.3 g/cm^3; weight $W = 150.5$ g; and layer thickness of 8.0 cm. The underlayer consisted of stones of nominal size $D_{n50} =$ 2.0–2.5 cm and thickness of 6.0 cm, whereas the core consisted of stones of nominal size $D_{n50} = 1.3$ cm. Three different slopes of structure were tested, i.e., $\cot\theta = 1.33$, 1.5, and 2.0, where θ = angle of structure slope measured from horizontal. Fig. 2 shows the cross section of the breakwater of 1:1.5 slope, which is similar to that used in the test of Van der Meer (1988). A little wave overtopping occurred when the significant wave height was greater than 18 cm. Tetrapods were placed in two layers. The upper layer Tetrapods were placed randomly on the regularly placed lower layer. Because the Tetrapods in contact with the sidewalls of the flume have less degree of interlocking, they were fixed not to move and were not included in the calculation of damage. For $\cot\theta = 1.33$, 1.5, and 2.0, the numbers of Tetrapods were 380, 399, and 456 and the slope areas were 0.585, 0.609, and 0.709 m^2, respectively, so that the packing density was 1.05.

To cover both plunging and surging waves, the wave steepness varied between 0.012 and 0.056. Four different mean wave periods were applied: T_z= 1.36, 1.67, 2.10, and 2.45 s. For each wave period, five different significant wave heights ranging from 9 to 19 cm were used. Therefore, 20 tests were conducted for each different slope angle, which resulted in a total of 60 tests.

The wave measurement was made during the test on the flat bottom before the sloping beach as shown in Fig. 1. However, one needs to know the wave height at the toe of the breakwater. To obtain the relationship between the wave heights at the two places, wave measurements were made there before the breakwater was installed. A linear relationship was obtained between the wave heights for each wave period. This relationship was used in the conversion of the measured wave height on the flat bottom to that at the toe of the breakwater.

The modified Bretschneider-Mitsuyasu spectrum (Goda 2010) was used, which is equivalent to the Pierson-Moskowitz spectrum:

[1]Professor, Dept. of Civil and Environmental Engineering, Seoul National Univ., Seoul 151-744, Rep. of Korea (corresponding writer). E-mail: kdsuh@snu.ac.kr

[2]Graduate Student, Dept. of Civil and Environmental Engineering, Seoul National Univ., Seoul 151-744, Rep. of Korea. E-mail: jsriver@snu.ac.kr

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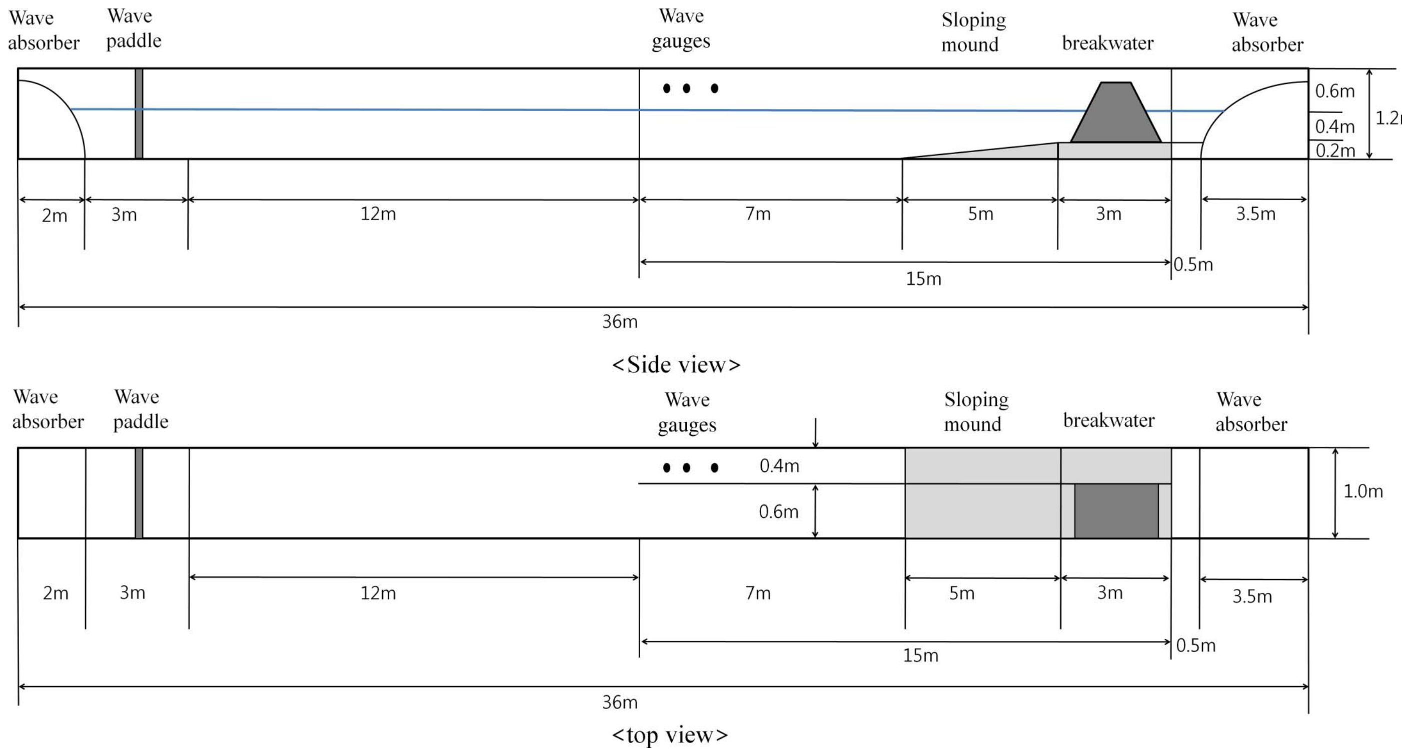

Fig. 1. Sketch of wave flume and experimental setup

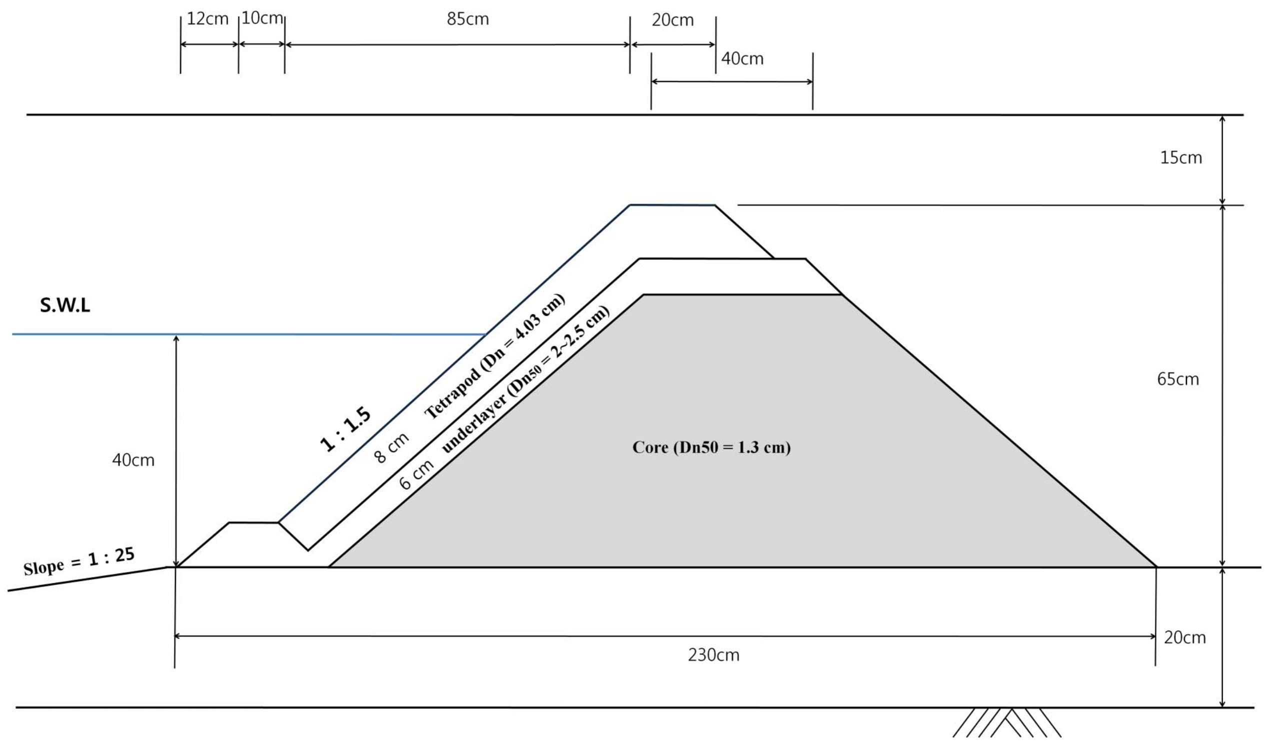

Fig. 2. Cross section of breakwater ($\cot\theta = 1.5$)

$$S(f) = 0.205\, H_s^2 T_s^{-4} f^{-5} \exp[-0.75(T_s f)^{-4}] \qquad (1)$$

where $S(f) =$ wave spectral density function; f = frequency; H_s = significant wave height; and T_s = significant wave period. The mean wave period was calculated by using the relationship $T_z = T_s/1.2$.

Each complete test consists of a short generation of small waves for stabilizing the Tetrapods, a test of 1,000 waves, an intermediate photographing, a test of 2,000 more waves, and a final photographing. The location and time of the displacing Tetrapods were visually observed during the test. A Tetrapod was regarded to be displaced when it moved more than one diameter, when it came back to its position after short displacement, or when it rotated more than 180°. After each complete test the armor layer was removed and rebuilt.

Derivation of Stability Formula

The test results are summarized in Table 1, in which N_0 = relative damage, which is the number of displaced armor units in a width (along the longitudinal axis of the breakwater) of one nominal size, and N = number of waves. The significant wave height and period were calculated by applying the zero-crossing method to the incident wave profile obtained from the wave separation.

The derivation of stability formula closely follows the procedure described in Van der Meer (1987a). First, by using the data in Table 1, damage curves are drawn for each wave period and storm duration (i.e., number of waves). An example of such damage curves is shown in Fig. 3, in which $\Delta = \rho_a/\rho_w - 1$ is the relative mass density of Tetrapods and ρ_w is the mass density of water. From these damage curves, the values of $H_s/\Delta D_n$ are taken for N_0 of 0, 0.5, and 1.5, and the corresponding surf similarity parameters are calculated. The surf similarity parameter is given by $\xi_z = \tan\theta/\sqrt{H_s/L_0}$, in which $L_0 = gT_z^2/(2\pi)$. The data set is augmented with the data of Van der Meer (1987b) and De Jong (1996) of the similar crest elevation and packing density to the present test.

Fig. 4 shows the influence of the surf similarity parameter and slope angle on the stability number, $N_s = H_s/\Delta D_n$. Results are shown for the damage $N_0 = 0.5$ after a wave attack of 1,000 waves. The curves are the stability formula that is derived subsequently in the paper. Note that $\cot\theta = 1.5$ for the data of Van der Meer (1987b) and De Jong (1996). There are six figures similar to Fig. 4 for different combinations of N_0(=0.0, 0.5, and 1.5) and N(=1,000 and 3,000), which are used for the derivation of the formula.

To obtain the formula consistent with those proposed by Van der Meer (1988) and De Jong (1996), the stability formula is expressed in terms of the following dimensionless variables: $H_s/\Delta D_n$, ξ_z, $\cot\theta$, and $N_0/\sqrt{N}$. Two stability formulas are proposed, one for plunging waves and one for surging waves.

For plunging waves on the left side of Fig. 4, the surf similarity parameter, ξ_z, describes the influence of slope angle and wave steepness on stability. These influences can be described by a power function: $H_s/\Delta D_n = a_1\xi_z^{b_1}$ with $a_1 = f(N_0/\sqrt{N})$. The coefficient b_1 is determined by a regression analysis for each of the six cases and the average value is approximately -0.4. Assuming $a_1 = a_2N_0^{0.5}/N^{0.25} + a_3$, the coefficient a_3 is determined by a regression analysis for the cases of $N_0 = 0.0$ and the average value is 3.25. Now the stability formula can be written as $H_s/\Delta D_n = (a_2N_0^{0.5}/N^{0.25} + 3.25)\xi_z^{-0.4}$. To determine the coefficient a_2 by a regression analysis, both the stability numbers read off from the damage curves for constant damage levels and the

Table 1. Test Results

Slope	T_z (s)	H_s (cm)	N_0		Slope	T_z (s)	H_s (cm)	N_0	
			$N = 1{,}000$	$N = 3{,}000$				$N = 1{,}000$	$N = 3{,}000$
1:4/3	1.36	9.4	0.074	0.074	1:1.5	2.08	12.5	0.004	0.004
	1.35	12.4	0.221	0.221		2.11	13.8	0.074	0.074
	1.36	14.2	0.442	0.516		2.09	14.7	0.074	0.147
	1.36	15.6	1.032	1.475		2.08	15.7	0.221	0.442
	1.40	17.3	1.770	3.466		2.05	17.6	0.369	1.549
	1.66	8.9	0.004	0.004		2.53	12.6	0.004	0.004
	1.64	12.2	0.004	0.221		2.51	14.3	0.074	0.074
	1.65	13.4	0.004	0.369		2.49	15.1	0.074	0.221
	1.66	14.4	0.295	0.959		2.34	18.7	0.737	2.434
	1.68	16.1	0.737	5.383		2.36	19.2	0.959	2.065
	2.10	12.4	0.074	0.074	1:2	1.32	8.7	0.004	0.004
	2.13	13.6	0.074	0.147		1.34	11.6	0.004	0.147
	2.14	15.9	0.295	0.516		1.34	13.3	0.295	0.516
	2.16	16.9	0.885	1.917		1.36	14.9	0.221	0.590
	2.15	18.7	1.991	5.162		1.39	16.6	0.885	2.876
	2.57	12.3	0.074	0.074		1.66	8.9	0.004	0.004
	2.53	14.0	0.369	0.442		1.68	11.6	0.004	0.147
	2.52	14.8	0.074	0.369		1.69	12.9	0.074	0.221
	2.36	16.9	2.212	3.614		1.69	13.9	0.074	0.221
	2.31	17.7	1.475	5.752		1.69	15.4	0.590	1.106
1:1.5	1.34	9.2	0.074	0.074		2.08	11.9	0.004	0.004
	1.36	12.3	0.147	0.369		2.11	13.2	0.004	0.004
	1.36	14.0	0.147	0.959		2.10	14.0	0.004	0.147
	1.36	15.6	0.590	1.106		2.09	14.9	0.295	0.442
	1.41	17.5	0.811	1.696		2.06	16.8	0.442	0.885
	1.67	9.0	0.004	0.004		2.50	11.7	0.074	0.147
	1.65	12.2	0.004	0.147		2.49	13.4	0.004	0.004
	1.67	13.5	0.295	0.516		2.48	14.2	0.147	0.369
	1.67	14.5	0.221	0.442		2.37	17.0	0.664	1.032
	1.69	16.1	0.885	1.991		2.31	17.2	0.590	1.180

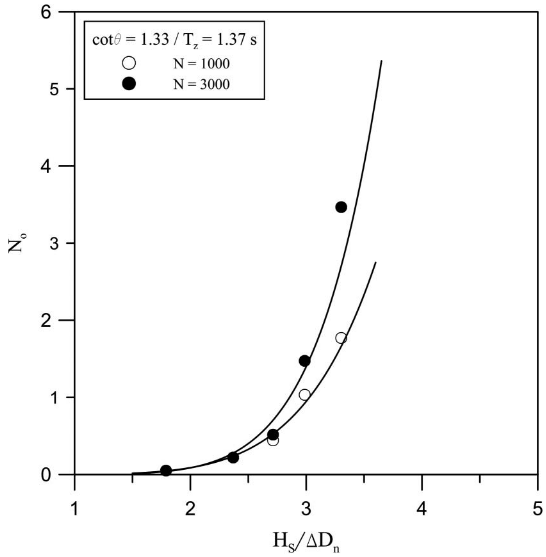

Fig. 3. Example of damage curve for $\cot\theta = 1.33$ and $T_z = 1.37\ s$

measured values (i.e., the data in Table 1) are used. The average value of a_2 is 9.2.

For surging waves on the right side of Fig. 4, different curves are shown for different slope angles. The procedure similar to plunging waves can be followed for surging waves, although the surf similarity parameter does not cover the influence of the slope angle. The influence of the wave steepness is described by $H_s/\Delta D_n = a_1\xi_z^{b_1}$ with $a_1 = f(N_0/\sqrt{N}, \cot\theta)$. For surging waves, 18 values of b_1 by regression analyses were obtained because different values of b_1 are determined for different slope angles. The average value of b_1 is approximately 0.4. The influence of the slope angle can be described by $H_s/\Delta D_n = a_2(\cot\theta)^{b_2}\xi_z^{0.4}$ with $a_2 = f(N_0/\sqrt{N})$. The coefficient b_2 is determined by a regression analysis, and the average value is 0.45. Expressing $a_2 = a_3 N_0^{0.5}/N^{0.25} + a_4$ and following the same procedure as plunging waves, the coefficients a_3 and a_4 are determined as 5.0 and 0.85, respectively.

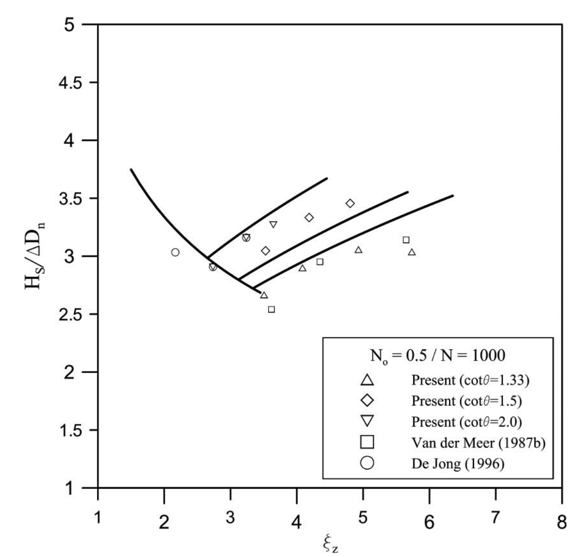

Fig. 4. Stability number versus surf similarity parameter in the case of $N_0 = 0.5$ and $N = 1{,}000$

Finally, the stability formula is proposed as

$$\frac{H_s}{\Delta D_n} = \max\left[\left(9.2\frac{N_0^{0.5}}{N^{0.25}} + 3.25\right)\xi_z^{-0.4},\ \left(5.0\frac{N_0^{0.5}}{N^{0.25}} + 0.85\right)(\cot\theta)^{0.45}\xi_z^{0.4}\right] \quad (2)$$

The preceding equation is basically the same as those of Van der Meer (1988) and De Jong (1996), except that a slope angle term is included in the surging wave formula and the first coefficients (i.e., 9.2 and 5.0) are a little larger.

It may be possible to derive a formula that represents the transition from plunging to surging waves. However, one could compare the formulas and take the one that gives the higher stability number.

De Jong (1996) proposed a formula similar to the plunging wave formula in Eq. (2), but De Jong's formula contains additional terms that take into account the influence of crest elevation and packing density. Borrowing these terms, Eq. (2) can be written as

$$\frac{H_s}{\Delta D_n} = \max\left[\begin{array}{l}\left(9.2\frac{N_0^{0.5}}{N^{0.25}} + 3.25f(\phi)\right)\xi_z^{-0.4}f(R_c/D_n),\\ \left(5.0\frac{N_0^{0.5}}{N^{0.25}} + 0.85f(\phi)\right)(\cot\theta)^{0.45}\xi_z^{0.4}f(R_c/D_n)\end{array}\right] \quad (3)$$

where

$$f(\phi) = 0.40 + 0.61\phi/\phi_{SPM} \quad (4)$$

$$f(R_c/D_n) = 1 + 0.17\exp(-0.61R_c/D_n) \quad (5)$$

$\phi =$ packing density, the normal value of which is 1.02; ϕ_{SPM} = packing density given in the Shore Protection Manual (U.S. Army 1984), which is 1.04 for Tetrapods; and R_c = crest elevation of the breakwater.

Comparison of Proposed Formula with Test Results

Fig. 5 compares the measured stability number with the prediction by Eq. (2) for the test results of the present study and other

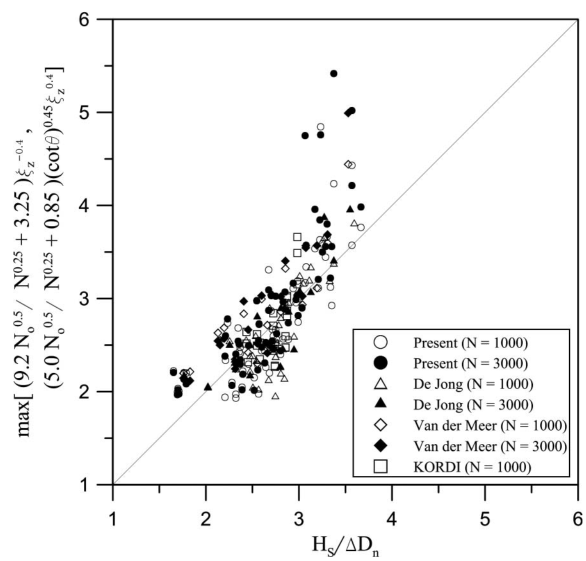

Fig. 5. Comparison of stability number between measurement and prediction by Eq. (2) for all available data excluding low-crested breakwaters and lower packing density of De Jong (1996)

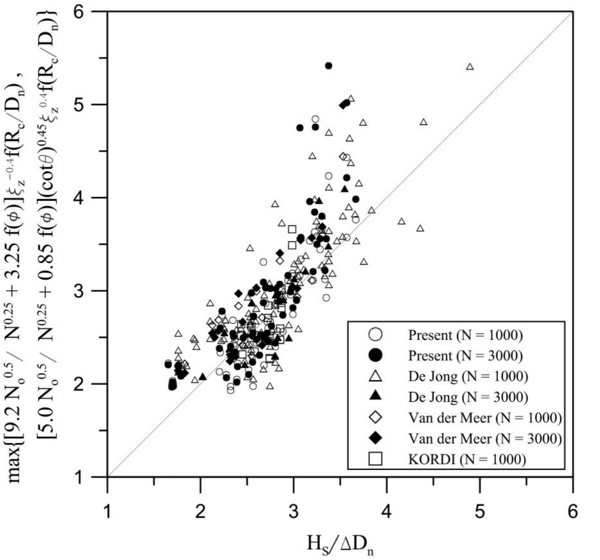

Fig. 6. Comparison of stability number between measurement and prediction by Eq. (3) for all available data including low-crested breakwaters and lower packing density of De Jong (1996)

researchers: Van der Meer (1987b); De Jong (1996), and Korea Ocean Research and Development Institute (KORDI 2001). Only the data of high-crested breakwaters and normal packing density of De Jong (1996) were used. The data of KORDI was for a breakwater of the slope angle of 1:1.33. The index of agreement is 0.843. The index of agreement is a statistical parameter proposed by Willmott (1981) as a measure of the degree to which a model's predictions are error-free. It varies between 0 and 1.0, where 1.0 indicates perfect agreement between observation and prediction, and 0 connotes complete disagreement. Fig. 6 compares the measured stability number with the prediction by Eq. (3). The data of low-crested breakwaters and lower packing density of De Jong (1996) were additionally included. The index of agreement is 0.857, indicating that Eq. (3) including additional terms for crest elevation and packing density is slightly more accurate than Eq. (2). All the overpredicted data points located around the predicted stability number of 5.0 in Figs. 5 and 6 are the cases of $\cot\theta = 1.33$, $H_s/\Delta D_n > 3.0$, and $N = 3{,}000$ except one case in which $N = 1{,}000$. This implies that the developed formula overpredicts the stability number in the case where a steep breakwater is designed for a storm of large wave height and long duration.

Uncertainty of Proposed Formula

Since the 1980s, reliability-based design methods have been developed for breakwater designs. Most of the design formulas have been developed based on hydraulic model test results, and the uncertainty of the formula is important in the reliability-based design. The stability formula proposed in this study can be written in the form of a reliability function:

$$G = \{a\Delta D_n[9.2N_0^{0.5}/N^{0.25} + 3.25f(\phi)]f(R_c/D_n) - H_s\xi_z^{0.4} \text{ or } a\Delta D_n[5.0N_0^{0.5}/N^{0.25} + 0.85f(\phi)](\cot\theta)^{0.45}f(R_c/D_n) - H_s\xi_z^{-0.4}\} = 0 \quad (6)$$

where a = variable signifying the uncertainty inherent in the formula. The uncertainty of a random variable is best given by a probability distribution. However, because the true distribution is rarely known, it is common to assume a normal distribution and a related coefficient of variation defined as

$$\sigma' = \frac{\sigma}{\mu} \quad (7)$$

as the measure of the uncertainty, where μ and σ are the mean and standard deviation, respectively. To calculate these statistical characteristics of the variable a, it was expressed as

$$a = \frac{H_s\xi_z^{0.4}}{\Delta D_n\left[9.2\frac{N_0^{0.5}}{N^{0.25}} + 3.25f(\phi)\right]f(R_c/D_n)} \quad \text{or} \quad \frac{H_s\xi_z^{-0.4}}{\Delta D_n\left[5.0\frac{N_0^{0.5}}{N^{0.25}} + 0.85f(\phi)\right](\cot\theta)^{0.45}f(R_c/D_n)} \quad (8)$$

The mean, standard deviation, and coefficient of variation obtained using the data of high-crested breakwaters with normal packing density are 0.972, 0.134, and 0.138, respectively. These values are 0.957, 0.133, and 0.139, respectively, if we include the data of low-crested breakwaters and lower packing densities of De Jong (1996). Therefore, the bias and coefficient of variation of the proposed formula are −0.04 and 0.14, respectively.

Conclusion

We proposed a stability formula for Tetrapods armoring a rubble mound breakwater based on hydraulic model test results of the present and previous studies. The proposed formula was proven to be applicable to breakwaters with various slope angles. It was also shown to be applicable to low-crested breakwaters and different packing densities if the corresponding terms are incorporated in the formula. The proposed formula overpredicts the stability number in the case where a steep breakwater is designed for a storm of large wave height and long duration. The bias and coefficient of variation of the proposed formula are −0.04 and 0.14, respectively.

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